To Count or Not to Count

A Personal Perspective

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I.I.T. Bombay

A Short Talk at VardiFest 2022, Haifa

Some historical context ….

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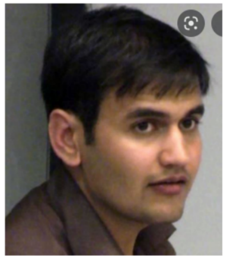

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    - Given propositional formula F with solution set Sol, params $e > 0$ and $0 < d < 1$
      - Generate a random y s.t.
        - $1/(1+ e) \times 1/|Sol| <= Pr[y \text{ is generated}] <= (1+e) \times 1/|Sol|$
      - Find a number C s.t.
        - $Pr[1/(1+e) \times C <= | Sol | <= (1+e) \times C] >= 1 - d$

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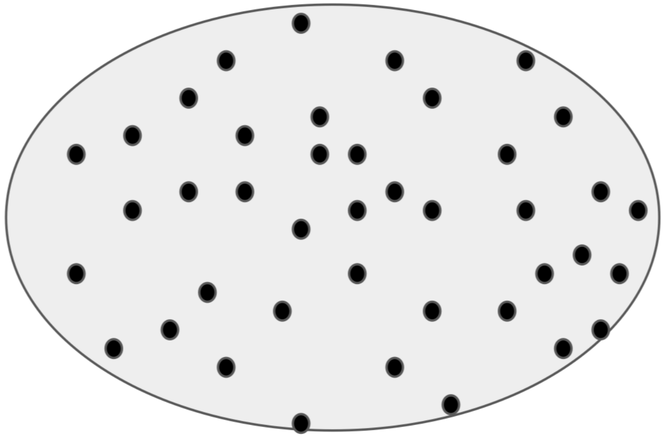
Moshe throws us a challenge:

Can we marry rigorous approximation guarantees with practical scalability for counting & sampling?

The Key Idea



Solution space of F

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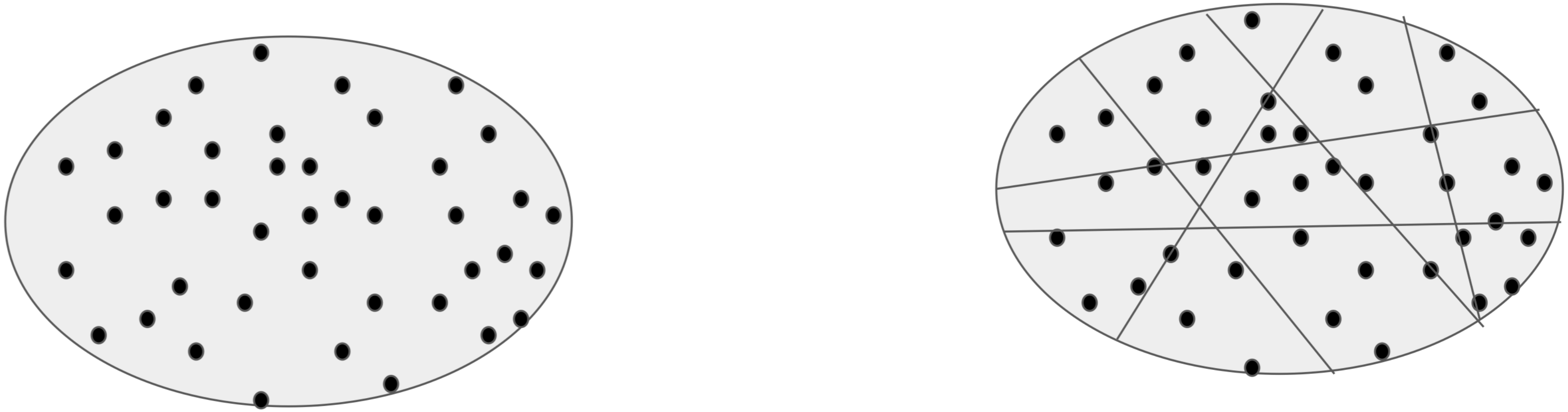

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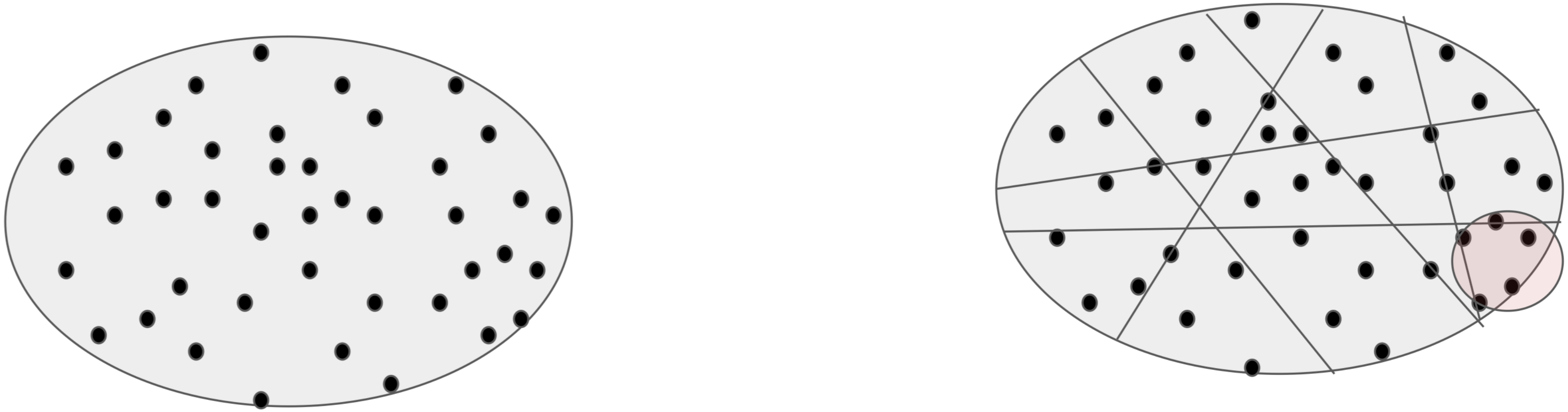

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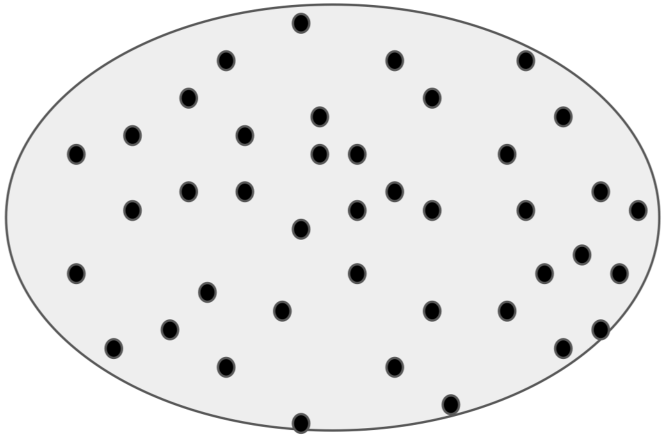

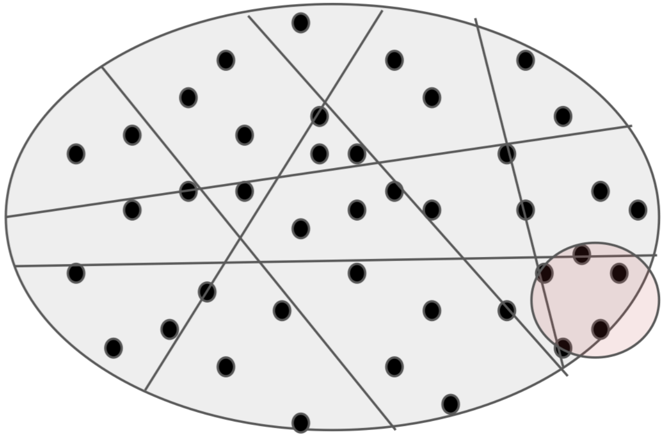

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Sample/count from this “small” cell, and scale if needed to lift to original domain

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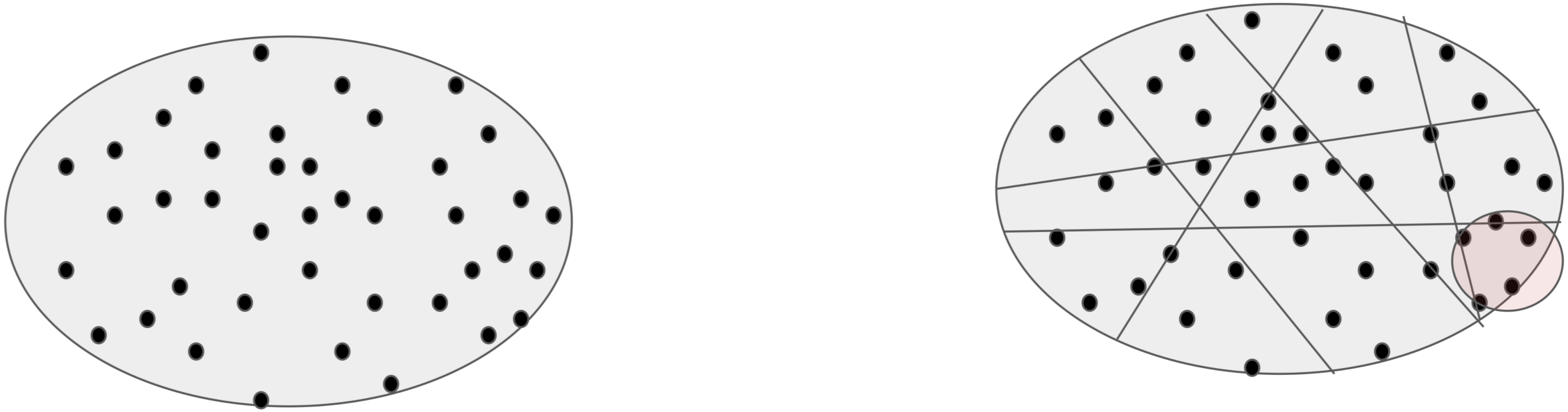

r-independent universal hash functions: **How small can r be?**

As r increases, stronger theoretical guarantees, but scalability setback

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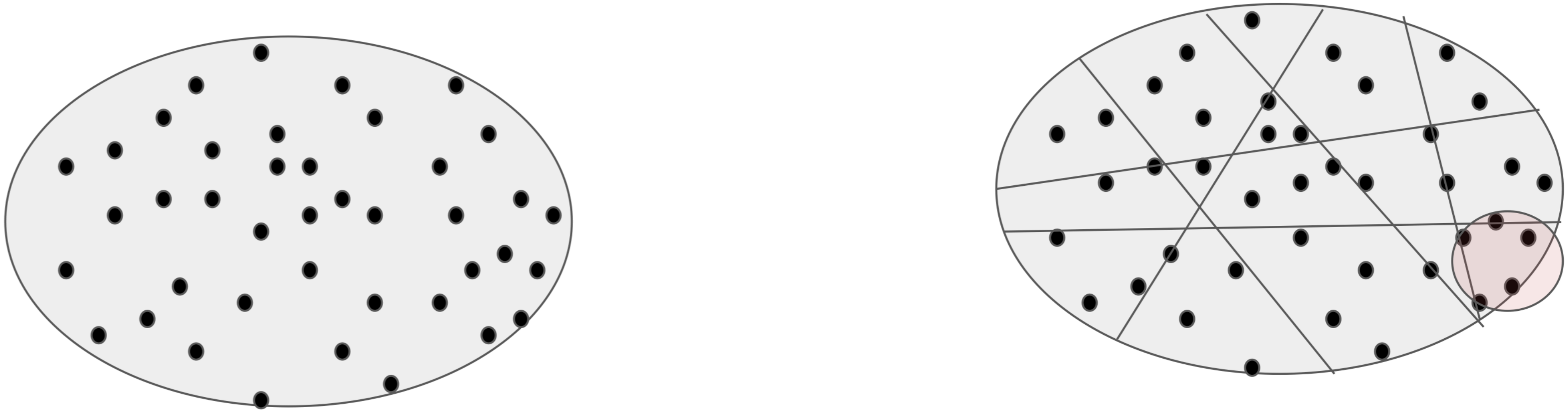

Moshe’s insight: r somewhere between 2 and 3 should work

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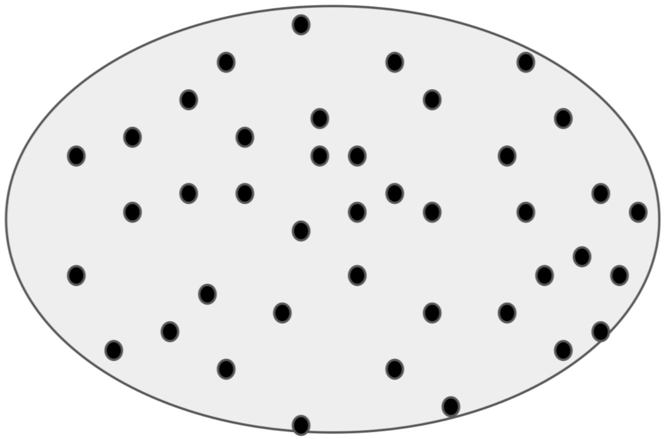

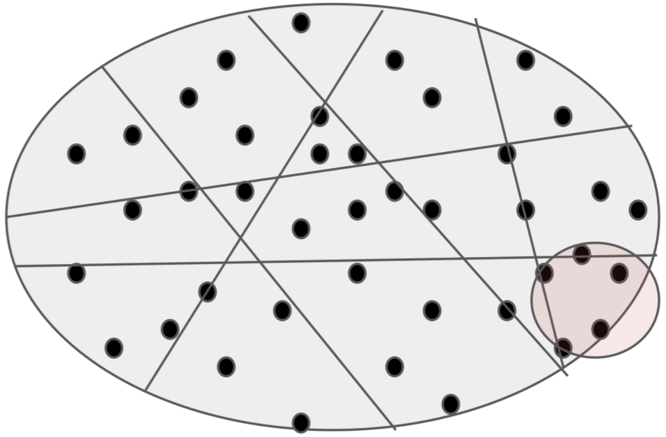

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Kuldeep’s thesis showed that indeed Moshe was right.

Can marry rigorous guarantees (PAC) with scalability (~ 10^6 vars)

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  - From my student days
  - Hard to put any quantitative measure on it
- Certain things are best left not counted

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- All the different things that I learnt (and continue to learn) from Moshe
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  - Hard to put any quantitative measure on it
- Certain things are best left not counted
- A small (not at all uniform) sample of a few results that have left deep impact on me

Automata Theoretic Verification

My graduate school days:

- Moshe Vardi and Pierre Wolper, “An Automata Theoretic Approach to Automatic Program Verification”, LICS 1986
    - “... for any temporal formula we can construct an automaton that accepts precisely the computations that satisfy the formula. The model-checking algorithm that results from this approach is much simpler and cleaner than tableau-based algorithms…”

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- My canonical example of how boundaries between theoretical and applied Computer Science are best broken

Logic and Finite Model Theory

My mid-career days, advising my student working on logic:

- Ron Fagin, “Generalized first-order Spectra and Polynomial-time Recognizable Sets”, Complexity of Computation 1973
- Moshe Vardi, “The Complexity of Relational Query Languages”, STOC 1982
    - Introduction of data complexity and query complexity of logical languages
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- Opened up new connections between logic, finite structures and complexity
- The inspiration from these and related papers led my student to complete his Ph.D. dissertation on logic and model theory in 2016

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Thank you, Moshe!

Priceless Moments – No Algo can Count Their Value

Moshe and Phokion with my 2-year old daughter in Dec 2011